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Stability Issues of the Mu2e Proton Beam

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Abstract

Stability issues of the mu2e proton beam are discussed. These include space-charge distortion of bunch shape, microwave instabilities, mode-coupling instabilities, head-tail instabilities, as well as electron-cloud effects.

INTRODUCTION

The proton beam destined to hit a target to produce pions which decay into muons for the mu2e experiment has to meet some very stringent specifications. The proton bunches originated in the Fermilab Booster are coalesced in the Recycler into four bunches, which are stored in the Accumulator, and transferred one by one to the Debuncher, where extraction is made in slow spills to hit the target [1]. The purpose of this article is to study the beam along the various rings in the collective-instability aspects [2]. Some properties of these three rings are listed in Table. I.

Table I: Circumferences C, transition gammas γ_t, and the rf voltages and harmonics V_i/h_i of the three Fermilab rings.

	Recycler	Accumulator	Debuncher
$C=2\pi R$ (m)	3319.418	474.098	505.283
γ_t	19.968	6.549	7.640
V_i/h_i (kV)	80/28, $-16/56$	100/4	30/4

BUNCH SHAPE DISTORTION

When each intense bunch is formed by coalescing 21 Booster bunches in the Recycler, strong space charge can distort the bunch shape, leading to bunch lengthening. The trajectory in the longitudinal phase space that passes through the energy offset $\Delta E=0$ and time advance $\tau=\sigma_\tau$, the rms bunch length, and the area it encloses are given by

$$(\Delta E)^2=\frac{2\beta^2E}{\eta}\left\{\sum_i\frac{eV_i}{2\pi h_i}\Big[\cos(\phi_s-h_i\omega_0\tau)\right.$$
$$\left.-\cos(\phi_s-h_i\omega_0\sigma_\tau)\Big]-\frac{e^2N_b}{2\pi}\left|\frac{Z_\parallel}{n}\right|\Big[\rho(\tau)-\rho(\sigma_\tau)\Big]\right\},\quad(1)$$

$$\sigma_A=\int_0^{\sigma_\tau}d\tau\left\{\frac{32\beta^2E}{\eta}\left[\sum_i\frac{eV_i}{2\pi h_i}\Big[\cos(\phi_s-h_i\omega_0\tau)\right.\right.$$
$$\left.\left.-\cos(\phi_s-h_i\omega_0\sigma_\tau)\Big]-\frac{e^2N_b}{2\pi}\left|\frac{Z_\parallel}{n}\right|\Big[\rho(\tau)-\rho(\sigma_\tau)\Big]\right]\right\}^{\frac{1}{2}}.\quad(2)$$

In above, $E=8.9383$ GeV is the nominal beam energy, β and γ are the nominal relativistic parameters, η is the slip parameter, $N_b=1\times10^{12}$ is the number per bunch, $Z_\parallel/n$ is the space charge impedance, and ϕ_s is the synchronous phase. Here the only unknown is the linear distribution function $\rho(\tau)$, which can be approximated by a Gaussian. We see that the space-charge force counteracts/enhances the rf force below/above transition. As a measure of the space-charge effect, we introduce the parameter

$$f_{\rm spch}=\frac{\text{linear spch}}{\text{linear rf}}=\frac{\frac{e^2N_b|Z_\parallel/n|}{2\pi}\frac{1}{\tau}\frac{d\rho}{d\tau}}{\sum_i\frac{eV_i}{T_0}h_i\omega_0}.\qquad(3)$$

For Gaussian distribution near $\tau=0$, $f_{\rm spch}=eN_b|Z_\parallel/n|/(\sqrt{2\pi}\sum_iV_ih_i\omega_0^2\sigma_\tau^3)$. Assuming no blowup, the transverse normalized emittance of the coalesced bunch in the Recycler is $\epsilon^n_{\rm rms}=6.5$ πmm-mr. Using the mean vertical betatron function of $\bar\beta_y=21.6$ m for the Recycler, the vertical beam radius is 3.85 mm. The vertical radius of the beam pipe is 2.22 cm. The space charge impedance is $Z_\parallel/n=i7.03$ Ω. The full bunch area in Booster is 0.1 eVs. If the coalescence is perfect without phase-space dilution, each of these 4 bunches will have an rms area of $21\times0.1/6=3.5$ eV-s. The matched rms length is $\sigma_\tau=17.7$ ns after solving Eq. (2). We obtain $f_{\rm spch}=0.0783$, which is not too small. However, the present coalescence scheme does enlarge the bunch area. If the enlargement is 3-fold, the rms bunch length becomes $\sigma_\tau=31.5$ ns while the space-charge parameter decreases to $f_{\rm spch}=0.021$, which is pretty small.

MICROWAVE INSTABILITIES

According to the properties of the 3 rings listed in in Table I, the Keil-Schnell limits of microwave instability are computed. The results are listed in Table II, where we assume the bunch area increases by 3 or 5 times after coalescence in the Recycler. The coupling impedances of the Recycler and the Accumulator have been studied in detail [3, 4], and are much smaller than the computed limits. The Debuncher impedance has not been fully studied, but its large vacuum-chamber aperture, ~10 cm radius, should warrant a small impedance. In short, the coalesced proton bunches should be safe against microwave instability.

ELECTRON CLOUD

We run POSINST for the proton bunches in the Accumulator [5]. The electron density averaged within the one sigma ellipse of the bunch is shown in Fig. 1 as a function of rf bucket number. A peak secondary electron yield of SEY$=2$ has been used. Superimposed is the electron-

Table II: Matched longitudinal bunch sizes and microwave stability limits of the $N_b = 1 \times 10^{12}$ bunch inside the Recycler, Accumulator, and Debuncher. The space-charge impedance is assumed to be $Z_\parallel/n = i7.03\ \Omega$ for the bunch in all three rings .

	σ_τ (ns)	σ_E (MeV)	$I_{\rm pk}$ (A)	$\lvert Z_\parallel/n\rvert$ limit (Ohms)
rms bunch area $= 1.05$ *eVs, blown-up 3 times*				
Recycler	31.52	10.63	2.028	333.1
Accumulator	19.90	16.81	3.212	760.8
Debuncher	23.27	14.39	2.747	323.8
rms bunch area $= 1.75$ *eVs, blown-up 5 times*				
Recycler	40.00	13.45	1.578	677.8
Accumulator	25.76	21.66	2.481	1635.
Debuncher	30.17	18.51	2.119	694.7

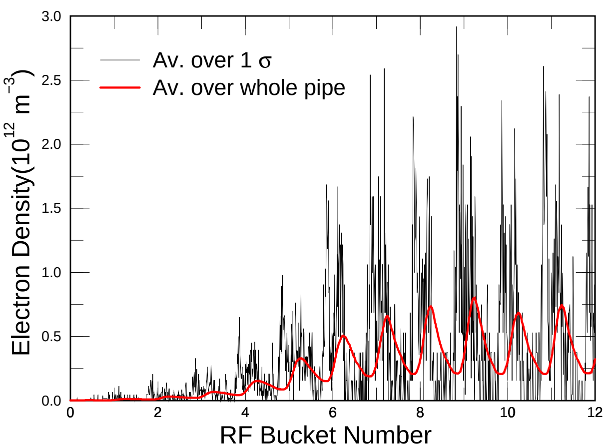

Figure 1: (Color) Electron cloud density generation in the Accumulator Ring as a function of rf bucket number. One revolution turn is 4 rf buckets. Black: averaged over electrons within one sigma ellipse of the bunch. Red: averaged over the whole beam pipe. SEY$=2.0$ has been assumed.

cloud density averaged over the whole vacuum chamber. Here we use for the transverse rms beam radii $\sigma_x/\sigma_y = 2.74/2.41$ mm and the rms bunch length $\sigma_\tau = 40$ ns. We see that it requires about the distance of 6 rf buckets or one-and-a-half revolution turns for the electron cloud to build up to the average density of $\rho_e \sim 2 \times 10^{12}$ m^{-3} when averaged over the one sigma ellipse of the bunch.

As for the Debuncher, there will only be a single bunch and the 3 consecutive empty rf buckets serves as a generous wide gap. Simulation shows that the electron density, built up in the presence of the bunch, falls back to zero in the gap. The electron density within one sigma of the beam averaged to only $\rho_e \sim 4 \times 10^{10}$ m^{-3}. The Recycler is 7 times as large as the Accumulator and Debuncher. The 4 coalesced proton bunches occupy 4 consecutive rf buckets in the $h = 28$ rf system. Thus there are 24 empty rf buckets for the electrons to subside before encountering the 4 proton bunches again in the next revolution turn. According to the above simulation of the Accumulator, electron cloud density can build up to $\rho_e \sim 3 \times 10^{11}$ m^{-3} only with 4 bunches when averaged over one sigma. We expect this to be the amount of electron density at the last proton bunch in the Recycler.

Two-Stream Oscillations

The electron cloud and the proton beam attract each other and can develop coupled two-stream oscillations, leading to increase of beam emittance. Landau damping comes from the betatron tune spread $\Delta\nu_y$ and the spread of the electron-bounce frequency $\Delta\omega_e$. The stability limit has been derived by Schnell and Zotter [6], which is

$$\frac{\Delta\nu_y}{\nu_y}\frac{\Delta\omega_e}{\omega_p} \gtrsim \frac{9\pi^2}{64}\frac{w_p^2}{\nu_y^2\omega_0^2}, \qquad (4)$$

with the electron-bounce frequency in the beam potential

$$\frac{\omega_e}{2\pi} = \frac{c}{2\pi}\sqrt{\frac{2N_b r_e}{\sigma_y(\sigma_x+\sigma_y)\sqrt{2\pi}\sigma_z}}, \qquad (5)$$

with $\sigma_z = \sigma_\tau\beta c$, and the proton-bounce frequency in the electron cloud potential in the absence of betatron focusing

$$\frac{\omega_p}{2\pi} \approx \frac{c}{2\pi}\sqrt{\frac{\pi\rho_e r_p}{\gamma}}, \qquad (6)$$

c being the velocity of light. In above, r_e and r_p are the electron and proton classical radii, and $\rho_e = 2\times 10^{12}$ m^{-3} is the electron density in one sigma of the beam obtained from the earlier simulation in the Accumulator. We obtain $\omega_e/2\pi = 186$ MHz and $\omega_p/2\pi = 48.0$ kHz. The two-stream stability limit becomes $\Delta\nu_y\Delta\omega_e/\omega_e \gtrsim 0.0009$ with vertical tune $\nu_y = 8.67$. The spread of electron-bounce frequency is usually quite large. Even if we take $\Delta\omega_e/\omega_e \sim 0.1$, a tune spread of $\Delta\nu_y \sim 0.01$ will be sufficient to damp the instability. For the Recycler/Debuncher, the electron-cloud density ρ_e will be one/two orders of magnitudes smaller. As a result, there will not be any chance for two-stream oscillation instabilities to develop.

Wake Fields

Electron cloud generates wake impedance which will affect the stability of the beam particles. The transverse electron-cloud impedance derived by Heifets is [7]

$$Z_1^\perp(\omega) = \frac{4Z_0\rho_e R}{\lambda_b^{\rm pk}\beta} Z_{\rm eff}\left(\frac{\omega}{\omega_e}\right) \qquad (7)$$

for a round beam, where the dimensionless reduced impedance $\mathcal{R}e\, Z_{\rm eff}$ rises slowly with frequency before reaching the electron-bounce frequency and drops abruptly to almost zero afterward. In above, $\lambda_b^{\rm pk} = N_b/\sqrt{2\pi}\sigma_z$ is the peak linear density and $Z_0 = 376.7\ \Omega$ is the free-space impedance.

TRANSVERSE MODE-COUPLING

Transverse mode-coupling instability (TMCI) occurs when two excitation modes of the bunch merge together. We compute these modes as functions of number per bunch N_b for the Accumulator driven by resistive-wall and electron-cloud impedances. The electron density at one sigma is taken as $\rho_e = 2\times10^{12}$ m^{-3} at $N_b = 1\times10^{12}$ and is assumed to vary linearly with N_b. Figure 2 shows the result with zero chromaticity, with 3 radial modes for each azimuthal mode number m. Although there is a lot of numerical noise, careful examination reveals that these excitation modes just cross each other without merging, and therefore no instability will develop.

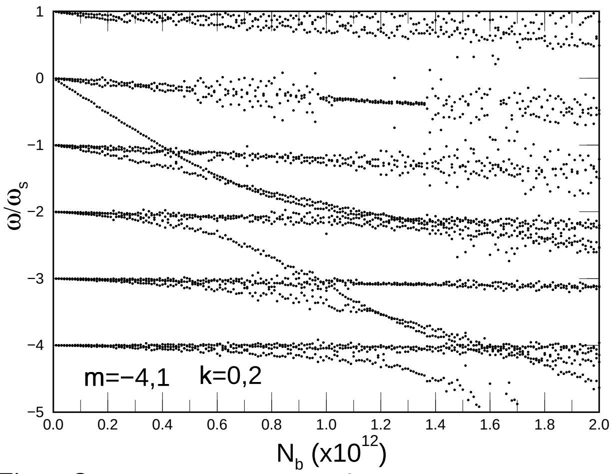

Figure 2: Transverse eigen-states of a bunch in the Accumulator as functions of bunch intensity N_b. No instability is observed.

HEAD-TAIL INSTABILITIES

Head-tail instabilities are driven by the transverse wake when the chromaticity $\xi_y \neq 0$. We concentrate on the proton bunches inside the Accumulator, where the electron density $\rho_e \sim 2\times10^{12}$ m^{-3} can be important near the electron-bounce frequency $\omega_e/2\pi = 186$ MHz. The resistive-wall impedance is also included.

Figure 3 shows the growth/damping rates of some lower azimuthal modes as functions of chromaticity ξ_y. Gaussian excitation modes are used. The spectra of all modes with $|m| \lesssim (\omega_e\sigma_\tau)^2 = 2190$ are in between $\pm\omega_e$ at $\xi_y = 0$. When ξ_y increases in the positive direction, these modes will be damped since their spectra overlap the electron-cloud impedance more at $+\omega_e$ than at $-\omega_e$. On the other hand, modes with $|m| \gtrsim 2190$ will become unstable. However, the power spectra of modes with such high azimuthal numbers have very tiny amplitudes and are therefore unimportant. The insert in Figure 3 shows a zoomed view for small ξ_y, where modes with $|m| > 1$ are unstable. They are driven by the resistive-wall impedance. The growth rates are less than ~ 5 s^{-1} and are unimportant, since these proton bunches reside in the Accumulator for at most 64 ms only. Nevertheless, they can be ameliorated completely by operating with $\xi_y \gtrsim 0.2$.

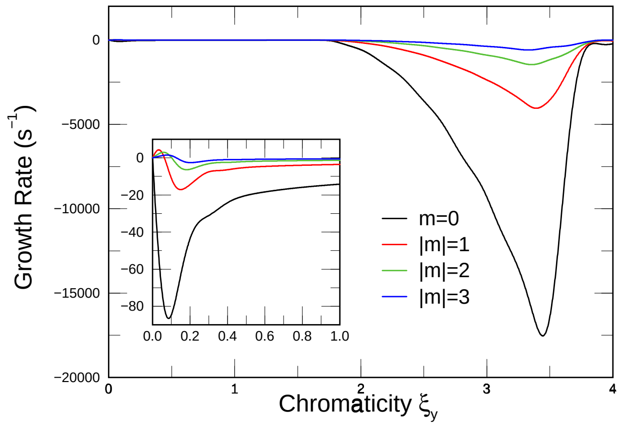

Figure 3: (Color) Head-tail growth rates for some lower azimuthal modes as functions of chromaticity ξ_y. Electron-cloud impedance dominates near $|\xi_y| \sim 3.5$, while resistive-wall impedance dominates when $|\xi_y|$ is small. The latter is zoomed in the insert.

CONCLUSION

We have studied several beam stability issues of the proton beam heading to the target for the mu2e experiment. We find bunch-shape distortions driven by the space charge force is reasonably small, and longitudinal microwave instability will unlikely to occur. Electron-cloud buildup, with density up to $\rho_e \sim 2\times10^{12}$ m^{-3} in the Accumulator, can probably drive head-tail instabilities. However, these, together with the instabilities driven by the resistive-wall impedance can be avoided by restricting the chromaticity to larger than ~ 0.2. TMCI will not occur even when the electron-cloud wake is included.

REFERENCES

[1] M. Syphers, *Possible Scheme to Ameliorate Space Charge and Momentum Spread Issues*, mu2e note, 2008.

[2] This is an extract of K.Y. Ng, Fermilab TM-2428-AD, 2009.

[3] K.Y. Ng, Fermilab-TM-2249, 2004.

[4] K.Y. Ng, Fermilab pbar note 470, 1976.

[5] M.A. Furman and G.R. Lambertson, Proc. Int. Workshop on Multibunch Instabilities in Future Electron and Positron Accelerators (MBI-97), ed. Y. H. Chin (KEK, Tsukuba, Japan, July 15–18, 1997).

[6] W. Schnell and B. Zotter, CERN Report ISR-GS-RF/76-26, 1976.

[7] S. Heifets, SLAC-PUB-9025, 2001.